Week 5 Lecture 1. 2003/02/05

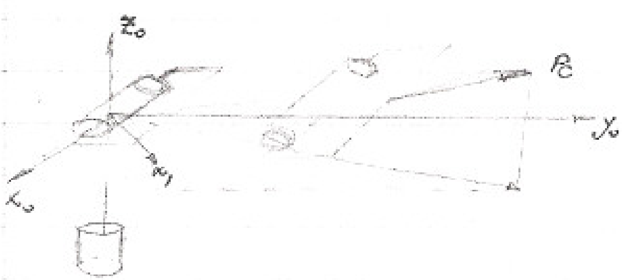
Kinematic Decoupling

Procedure

1. find $q_1, q_2, q_3$ such that the wrist centre $P_c$ is located at $P_c = d - R\begin{pmatrix} 0 \\ 0 \\ d_6 \end{pmatrix}$

2. using the joint variables found in step 1, evaluate ${}^0_3R$ (using forward kinematics)
3. find a set of Euler angles $q_4, q_5, q_6$ corresponding to the rotation matrix ${}^3_6R = {}^0_3R^T R$

Example 1.

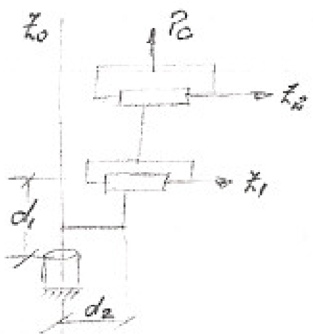



Given: $P_c = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$ coordinate of the wrist centre.

Find: $\theta_1, \theta_2, \theta_3$.

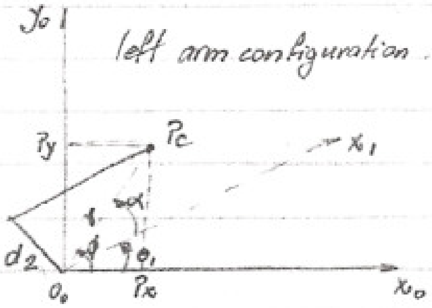
$\theta_1 = atan2(P_x, P_y)$
this solution is valid if $P_x \neq 0$ and $P_y \neq 0$
If $P_x = P_y = P_z \Rightarrow$ single configuration.
This means that we have infinite # of solutions for $\theta_1$.
This singularity occurs when the wrist centre is on the $z_0$ axis.

If there is an offset $d_2 \neq 0$.




The wrist centre $P_c$ cannot intersect $z_0$ axis.
In this case there will be only ~~the~~ two solutions for $\theta_1$. These correspond to the so-called left arm and right arm configurations.

left arm configuration.




$\theta_1 = \phi - \alpha$

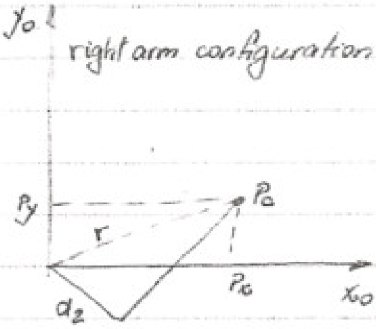
$\phi = \text{atan}2(P_x, P_y)$

$\alpha = \text{atan}2(\sqrt{r^2 - d_2^2}, d_2)$

$= \text{atan}2(\sqrt{P_x^2 + P_y^2 - d_2^2}, d_2)$

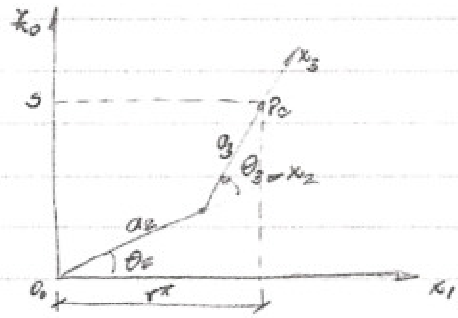
the second solution for $\theta_1$ is obtained using the right arm configuration.

right arm configuration.




$\theta_1 = \text{atan}2(P_x, P_y) - \text{atan}2(d_2, -\sqrt{P_x^2 + P_y^2 - d_2^2})$

Solution in the book has few mistakes corrected in the above example.




$\theta_2, \theta_3$ ?

$s = P_z - d_1$

$r^* = \sqrt{P_x^2 + P_y^2 - d_2^2} = \sqrt{r^2 - d_2^2}$

Cosine law : $c^2 = b^2 + a^2 - 2ab\cos(C)$

$\Rightarrow \cos\theta_3 = \dfrac{r^{*2} + s^2 - a_2^2 - a_3^2}{2a_2a_3} = \dfrac{P_x^2 + P_y^2 - d_2^2 + (P_z - d_1)^2 - a_2^2 - a_3^2}{2a_2a_3} = D.$

$-1 \leq D \leq 1$

$\theta_3 = \text{atan}2(D, \pm\sqrt{1 - D^2})$

Similarly, solve for $\theta_2$ : $\theta_2 = \text{atan}2(r^*, s) - \text{atan}2(a_2 + a_3c_3, a_3s_3)$

The two solutions are ~~added~~ corresponding to the elbow-up position and elbow down position.

Once $\theta_1$, $\theta_2$ & $\theta_3$ are obtained, we need to obtain ${}^0_3R$

We calculate ${}^3_6R = {}^0_3R^T R$

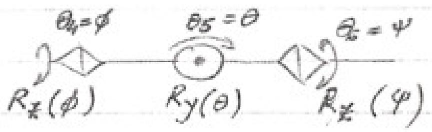
${}^0_6R$ (given.

for the spherical wrist we have

$$\begin{bmatrix} {}^3_6R & {}^3_6d \\ 0 & 1 \end{bmatrix} = A_4 A_5 A_6$$

$${}^3_6R = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 \\ S_4C_5C_6 + C_4C_6 & -S_4C_5C_6 + C_4C_6 & S_4S_5 \\ -S_5C_6 & S_5S_6 & C_5 \end{bmatrix} = \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix}$$

Solve for $\theta_4, \theta_5, \theta_6$.




$$\begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi c\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi s\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

known $\rightarrow$ ${}^3_6R = {}^0_3R^T \cdot R$.

Suppose that $u_{13}$ & $u_{23}$ are not both zero. then $s\theta \neq 0$.
and hence, not both of $u_{31}$ & $u_{32}$ are zero.

If ~~not~~ $u_{13}$ and $u_{23}$ are not both zero then $u_3 \neq \pm 1$ and we have $\underline{\underline{\cos\theta = u_{33}}}$

$$\sin\theta_3 = \pm\sqrt{1-u_{33}^2} \longleftarrow$$

$$\theta = \text{atan2}(u_{33}, \sqrt{1-u_{33}^2}) \quad \text{or} \quad \theta = \text{atan2}(u_{33}, -\sqrt{1-u_{33}^2})$$

if we choose the first value for $\theta$, then $\sin\theta > 0$ and

$$\phi = \text{atan2}(u_{13}, u_{23}) \longleftarrow$$

$$\psi = \text{atan2}(-u_{31}, u_{32}) \longleftarrow$$

$$\frac{s\theta s\psi}{-s\phi c\psi} = \frac{u_{32}}{u_{31}} \implies \tan\psi = -\frac{u_{32}}{u_{31}}$$